Journal of Applied Mechanics and Technical Physics, Vol. 42, No. 2, pp. 279–283, 2001

NONSTATIONARY GAS FILTRATION CAUSED BY AN INTENSE THERMAL ACTION ON A DAMP POROUS MEDIUM

A. M. Vorob'ev,[1] **K. N. Egorov,**[2] **D. V. Eliseev,**[1]
V. V. Kozlov,[2] **and D. V. Sadin**[2]

UDC 532.546

A mathematical model and an algorithm are proposed for evaluating nonstationary heat and mass transfer in a porous medium that contains a mechanically absorbed liquid and a two-component gas (vapor–inert gas mixture). The case of an intense thermal action on a damp porous mixture caused by an external heat flux and convective heat transfer is considered. Typical flow regions and typical regions of the interaction between the phases are described.

1. Physical Model. We consider a damp porous medium with a structure shown in Fig. 1a. Each capillary consists of a chain of pores 1, which vary both in size and geometry, with slits (tubes) 2 of a much smaller cross-sectional area in between. The capillary contains mechanically absorbed water 3 and a vapor–inert gas mixture. The porous medium is bounded by a rigid impermeable wall with a heat flux Q suddenly added to the latter.

As the medium becomes more and more heated, the temperatures of all phases (solid "sponge," water, and gas) change, and phase transformations occur at the water–vapor interface. Variation of thermodynamic characteristics of the gas gives rise to a capillary flow (filtration) of the gas (and, in the general case, of the absorbed water) due to the pressure difference.

Gas filtration in a porous medium is determined by a force interaction between the gas and the solid "sponge." For the type of a porous medium under consideration, in which the pores are connected by relatively narrow slits, a substantial factor is the buoyancy force caused by the pressure gradient [1]:

$$\boldsymbol{F}_A = -\alpha_1 \nabla p(1-\varepsilon^2) \qquad (\varepsilon \ll 1). \tag{1}$$

Here α_1 is the porosity of the medium (ratio of the volume occupied by the gas to the total volume of the medium), p is the pressure, and ε is the ratio of the total cross-sectional area of the narrowest parts of slits at their exit from pores projected onto a given plane to the "free" area of this plane [1].

The forces of hydrodynamic friction caused by shear stress at the "sponge" surface can be represented in the form

$$\boldsymbol{F}_\mu = \frac{\alpha_1^2 \mu \boldsymbol{v}}{\chi}, \tag{2}$$

where μ is the dynamic viscosity of the gas, $\boldsymbol{v}$ is the velocity of the gas flow, and χ is a quantity having the dimension of area. If the porosity of the medium is spatially nonuniform, the force $-p\nabla\alpha_1$ should also be taken into account because of the stream-tube expansion.

---

[1]Specialized Mechanical-Engineering Design Office, St. Petersburg 194100. [2]Mozhaiskii Military Space-Engineering University, St. Petersburg 197082. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 42, No. 2, pp. 106–110, March–April, 2001. Original article submitted February 15, 2000; revision submitted October 12, 2000.

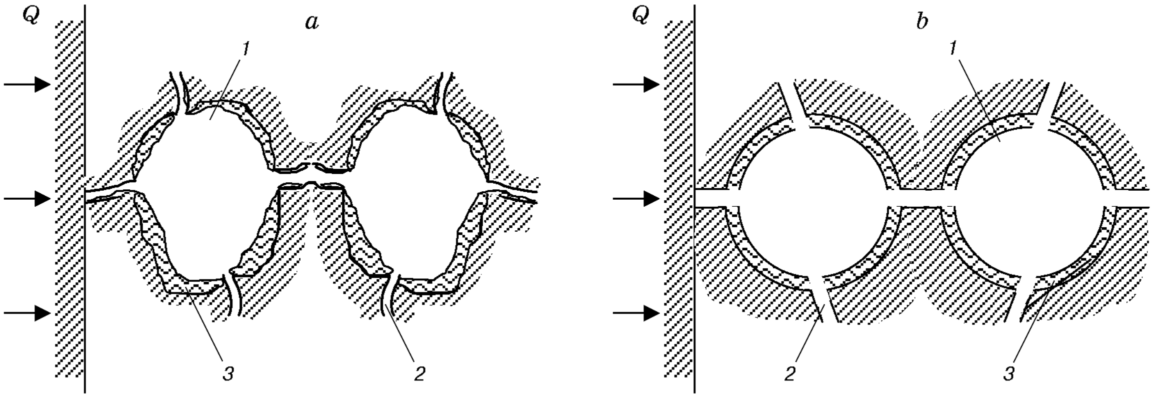

Fig. 1

For the conditions of interest, the gas flow through the porous medium has a wave character [2], i.e., the disturbances propagate with a finite velocity $a\varepsilon$ (a is the velocity of sound in the gas contained in the pores).

In the problem considered, the source of disturbances is an external pulsed thermal action exerted on the porous medium. In contrast [1, 2], we take into account both heat and mass transfer between the phases. Filtration processes are determined by the distribution of microparameters inside the pores. To simplify the problem, one should schematize both the medium structure (Fig. 1b) and the model of microprocesses. Following [3], instead of the temperature field in a pore, we use three characteristic temperatures: the temperature of the gas phase T_1, the temperature at the liquid–gas interface $T_{\rm s}$, and the temperature of the solid "sponge" T_3. The heat flux at the interface is specified by the Nusselt number, whereas the heat flux through the liquid film is determined by its thickness δ.

We assume that the gas phase consists of two components, an inert gas, which suffers no phase transitions, and a vapor. We also assume that no chemical reactions proceed between these components, and these components are calorically perfect gases with temperature-dependent thermal conductivities and additive thermophysical parameters.

Next, we assume that the pore diameters are many times greater than the characteristic molecular-kinetic lengths and many times smaller than the distances over which macroscopic characteristics vary appreciably. The pores are assumed to be spheres of identical diameters. The viscosity and thermal conductivity of the gas contained in the pores only affect the interaction between the phases. The gravity force is ignored.

2. Mathematical Model. Under the assumptions adopted, the heat and mass transfer in a damp porous medium is described by the following equations of conservation for the mass, momentum and energy of the phases:

$$\frac{\partial \rho_{\rm g}}{\partial t} + \nabla\cdot\rho_{\rm g}\boldsymbol{v} = 0, \qquad \frac{\partial \rho_{\rm v}}{\partial t} + \nabla\cdot\rho_{\rm v}\boldsymbol{v} = -J_{12}, \qquad \frac{dr_1}{dt} = -\frac{J_{12}r_1}{3\alpha_1\rho^0_{2\rm s}}, \qquad \frac{d\alpha_1}{dt} = -\frac{J_{12}}{\rho^0_{2\rm s}},$$

$$\rho_1\frac{d_1\boldsymbol{v}}{dt} = -\nabla p\alpha_1 - \boldsymbol{F} + J_{12}\boldsymbol{v}, \qquad \frac{d_1}{dt} \equiv \frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla,$$

$$\frac{\partial \rho_1 E_1}{\partial t} + \nabla\cdot\rho_1E_1\boldsymbol{v} + \nabla\cdot\alpha_1 p\boldsymbol{v} + p\frac{\partial\alpha_1}{\partial t} = Q_{\rm s1} - J_{12}i_{\rm v,s}, \qquad (3)$$

$$\frac{\partial T_3}{\partial t} = a_T^2\Delta T_3 + \frac{Q_{\rm s3}}{\rho_3c_3}, \qquad a_T^2 = \frac{\lambda_3}{\rho_3c_3}, \qquad \rho_i = \rho_i^0\alpha_i, \qquad \sum_{i=1}^{3}\alpha_i = 1, \qquad E_1 = u_1 + \frac{|\boldsymbol{v}|^2}{2}, \qquad i = 1,\,2,\,3.$$

Here the subscripts 1, 2, and 3 refer to the gas, liquid, and solid phases, respectively; the subscripts "g" and "v" refer to the inert gas and the vapor component, respectively, and the subscript "s" refers to the surface phase (Σ-phase); ∇ and Δ are the Hamilton and Laplace operators, ρ_i and ρ_i^0 are the reduced and true densities, respectively, α_i is the volume fraction of the ith phase, E_1 and u_i are the specific (per unit mass) total energy of the gas and the internal energy of the ith phase, respectively, $i_{\rm v,s}$ is the enthalpy of the vapor, which undergoes a phase transition at the interface between the phases, r_1 is the radius of a pore filled by the gas, J_{12}, $\boldsymbol{F}$, and $Q_{{\rm s},i}$ are, respectively, the rate of the phase transformation, the intensity of interaction between the phases, and the rate of heat transfer between the Σ-phase and the ith phase; a_T, λ_3, and c_3 are the thermal diffusivity, thermal conductivity, and heat capacity of the solid material, respectively.

Substituting the volume force acting between the phases [according to (1) and (2), $\boldsymbol{F} = \boldsymbol{F}_A + \boldsymbol{F}_\mu - p\nabla\alpha_1$] into the momentum equation for the gas phase (3) and re-writing the latter equation in a divergent form, we obtain

$$\frac{\partial \rho_1 \boldsymbol{v}}{\partial t} + \nabla \rho_1 \boldsymbol{v}\boldsymbol{v} + \alpha_1 \varepsilon^2 \nabla p = -\frac{\alpha_1^2 \mu_1 \boldsymbol{v}}{\chi}.$$

We calculate now the rate of heat and mass transfer between the phases using an equilibrium interface scheme [3]:

$$J_{12} l(p_{\rm v}) = Q_{\rm s1} + Q_{\rm s3}, \qquad Q_{\rm s1} = 1.5\,\frac{\alpha_1}{r_1^2}\,{\rm Nu}_1 \lambda_1 (T_{\rm s} - T_1),$$

$$p_2 = p - \frac{2\sigma}{r_1}, \quad p = p_{\rm g} + p_{\rm v}, \quad Q_{\rm s3} = 12\lambda_2\,\frac{\alpha_1 r_2}{\pi r_1^2 \delta}\,(T_{\rm s} - T_3).$$

Here $T_{\rm s}$ is the mean temperature at the liquid-phase surface, which is equal to the saturation temperature, $l(p_{\rm v})$ is the heat of evaporation, p_2 is the pressure in the liquid phase, σ is the surface tension, r_2 is the radius of a pore filled by the gas and the liquid, δ is the thickness of the liquid film at the pore walls, λ_i is the thermal conductivity of the ith phase, and ${\rm Nu}_1$ is the Nusselt number (${\rm Nu}_1 = 10$ being its quasistationary value for the internal heat-transfer problem).

System (3) of conservation equations is closed by the following equations of state for the calorically perfect gas components:

$$p_{\rm g} = \rho_{\rm g}^0 R_{\rm g} T_1, \quad p_{\rm v} = \rho_{\rm v}^0 R_{\rm v} T_1, \quad \rho = \rho_{\rm g} + \rho_{\rm v},$$

$$k_{\rm g} = \frac{\rho_{\rm g}}{\rho_1}, \quad k_{\rm v} = \frac{\rho_{\rm v}}{\rho_1} \quad (k_{\rm g} + k_{\rm v} = 1), \quad u_1 = k_{\rm g} u_{\rm g} + k_{\rm v} u_{\rm v}, \quad \lambda_1 = \lambda_1(k_{\rm g}, T_1),$$

$$i_{\rm g} = c_{\rm g}(T_1 - T^*) + i_{\rm g}^*, \qquad i_{\rm v} = c_{\rm v}(T_1 - T^*) + i_{\rm v}^*.$$

Here $p_{\rm g}$, $p_{\rm v}$ and $R_{\rm g}$ and $R_{\rm v}$ are the partial pressures and universal gas constants, respectively, $\rho_{\rm g}^0$ and $\rho_{\rm v}^0$, $k_{\rm g}$ and $k_{\rm v}$, and $u_{\rm g}$ and $u_{\rm v}$ are, respectively, the true densities, concentrations, and internal energies of the components (per unit mass), $c_{\rm g}$ and $c_{\rm v}$ are, respectively, the heat capacities of the inert gas and the vapor at constant pressure, and $i_{\rm g}$ is the enthalpy of the gas component; the superscript "$*$" denotes fixed parameters. The enthalpy of the vapor component $i_{\rm v}$ is related to the enthalpy of the condensed phase $i_{\rm liq}$ by the normalization condition

$$i_{\rm v}^* - i_{\rm liq}^* = l(p_{\rm v}^*) + (c_{\rm liq} - c_{\rm v})(T_{\rm s}(p_{\rm v}^*) - T^*),$$

where $c_{\rm liq}$ is the heat capacity of the liquid.

3. Calculation Procedure. The difference scheme for calculating nonstationary heat and mass transfer in a damp porous medium is constructed using splitting by physical processes in three stages. At the first stage, the heat-conduction equation is integrated by the sweep method [4]:

$$\frac{\partial T_3^{\rm I}}{\partial t} = a_T^2 \Delta T_3^{\rm I}. \tag{4}$$

At the second and third stages, the procedure follows the method of [5], in which interactions between the phases are taken into account implicitly at the Eulerian stage:

$$\frac{\partial \rho_{\rm g}^{\rm II}}{\partial t} = 0, \qquad \frac{\partial \rho_{\rm v}^{\rm II}}{\partial t} = -J_{12}, \qquad \frac{dr_1^{\rm II}}{dt} = -\frac{J_{12} r_1}{3\alpha_1 \rho_{\rm 2s}^0},$$

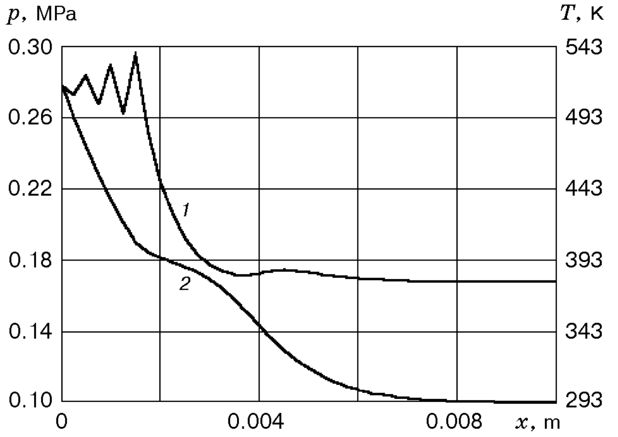

Fig. 2

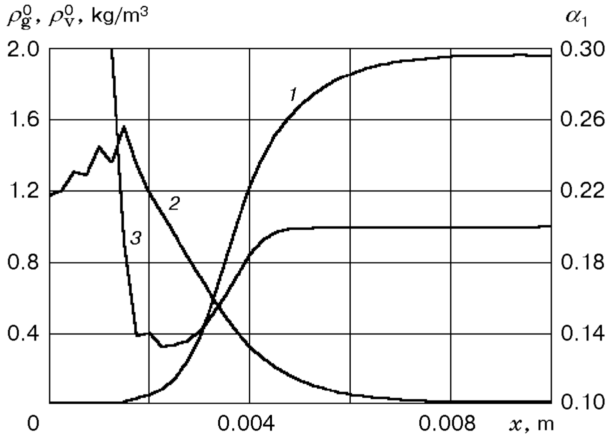

Fig. 3

$$\frac{d\alpha_1^{\mathrm{II}}}{dt} = -\frac{J_{12}}{\rho_{2\mathrm{s}}^0}, \qquad \rho_1 \frac{\partial \boldsymbol{v}^{\mathrm{II}}}{\partial t} + \alpha_1 \varepsilon^2 \nabla p = -\frac{\alpha_1^2 \mu_1 \boldsymbol{v}^{\mathrm{II}}}{\chi}, \tag{5}$$

$$\rho_1 \frac{\partial E_1^{\mathrm{II}}}{\partial t} + \nabla \cdot \alpha_1 p \boldsymbol{v} = Q_{\mathrm{s}1}^{\mathrm{II}} - J_{12} i_{\mathrm{v,s}}, \qquad \frac{\partial T_3^{\mathrm{II}}}{\partial t} = \frac{Q_{\mathrm{s}3}^{\mathrm{II}}}{\rho_3 c_3};$$

$$\frac{\partial \rho_{\mathrm{g}}^{\mathrm{III}}}{\partial t} + \nabla \cdot \rho_{\mathrm{g}} \boldsymbol{v}^{\mathrm{II}} = 0, \quad \frac{\partial \rho_{\mathrm{v}}^{\mathrm{III}}}{\partial t} + \nabla \cdot \rho_{\mathrm{v}} \boldsymbol{v}^{\mathrm{II}} = 0, \quad \frac{\partial (\rho_1 \boldsymbol{v})^{\mathrm{III}}}{\partial t} + \nabla (\rho_1 \boldsymbol{v}\boldsymbol{v})^{\mathrm{II}} = 0,$$

$$\begin{gathered} \frac{\partial (\rho_1 E_1)^{\mathrm{III}}}{\partial t} + \nabla \cdot (\rho_1 E_1 \boldsymbol{v})^{\mathrm{II}} + p \frac{\partial \alpha_1^{\mathrm{II}}}{\partial t} = 0, \\ r_1^{\mathrm{III}} = r_1^{\mathrm{II}}, \quad \alpha_1^{\mathrm{III}} = \alpha_1^{\mathrm{II}}, \quad T_3^{\mathrm{III}} = T_3^{\mathrm{II}}, \quad p_{\mathrm{g}}^{\mathrm{III}} = \rho_{\mathrm{g}}^{\mathrm{III}} R_{\mathrm{g}} (E_1^{\mathrm{III}} - (\boldsymbol{v}^{\mathrm{III}})^2/2), \end{gathered} \tag{6}$$

$$p_{\mathrm{v}}^{\mathrm{III}} = \rho_{\mathrm{v}}^{\mathrm{III}} R_{\mathrm{v}} (E_1^{\mathrm{III}} - (\boldsymbol{v}^{\mathrm{III}})^2/2), \qquad p^{\mathrm{III}} = p_{\mathrm{g}}^{\mathrm{III}} + p_{\mathrm{v}}^{\mathrm{III}}.$$

In (4)–(6), the superscripts designate stage numbers. Difference approximation is performed as in [5]. The scheme described above is a marching one, the time step τ should be chosen here from the Courant–Friedrichs–Lévy condition, which involves the ratio of the mesh step to the propagation velocity of weak disturbances $a\varepsilon$.

4. Initial Data. We consider a damp porous medium of thickness H bounded by impermeable walls. At the moment $t = 0$, the left wall experiences a pulsed thermal action of duration Θ caused by an external heat flux Q and convective heat transfer.

The problem is considered in a one-dimensional statement with the following initial parameters: $H = 0.01$ m, $Q = 600$ W/m^2, temperature of the right wall $T_{\mathrm{right}} = 293$ K, temperature of the left wall $T_{\mathrm{left}} = 1073$ K, initial volume fraction of the gas $\alpha_{10} = 0.2$, initial volume fraction of water $\alpha_{20} = 0.1$, $\lambda_3 = 1.7$ W/(m·K), $c_3 = 500$ J/(kg·K), $\rho_3 = 4000$ kg/m^3, $\varepsilon = 10^{-3}$, $\chi = 10^{-8}$ m^2, $\gamma_{\mathrm{g}} = 1.4$, heat-transfer coefficient $\alpha = 300$ W/(m$^2 \cdot$ K), $R_{\mathrm{g}} = 287$ J/(kg $\cdot$ K), $\Theta = 5$ sec, $\lambda_{\mathrm{g}} = 0.025$ W/(m $\cdot$ K), and $\mu_{\mathrm{g}} = 1.85 \cdot 10^{-5}$ Pa $\cdot$ sec.

To determine the thermophysical parameters of water and vapor, the tabulated data reported in [6] were used. All the fixed parameters of vapor for $T_1^* = 373$ K were borrowed from the tables of [6].

The following conditions were adopted for the initial moment: $T_1 = T_{\mathrm{s}} = T_3 = 293$ K, $p = 10^5$ Pa, and $\boldsymbol{v} = 0$. The partial pressures p_{g} and p_{v} can be found from the condition of phase equilibrium at $t = 0$. Both walls were assumed to be impermeable. The conditions for heat transfer at the left and right walls were $Q - \alpha(T_3 - T_{\mathrm{left}}) = -\lambda_3\, \partial T_3/\partial R$ and $\lambda_3\, \partial T_3/\partial R = \alpha(T_3 - T_{\mathrm{right}})$, respectively.

5. Some Results. As a result of the intense thermal action exerted on the left wall, the temperature of the "sponge" increases, and the water contained in it starts evaporating as its temperature rises. The pressure and temperature of the gas phase increase as well, which gives rise to its filtration. Figure 2 shows the distributions of the above-indicated parameters across the porous medium for $t = 5$ sec (curves 1 and 2 refer to the gas pressure and gas temperature, respectively). As is shown in Fig. 3 ($t = 5$ sec), the densities of the gas-phase components (curves 1 and 2 refer to air and vapor, respectively), which are spatially uniform and equilibrium at the initial moment, change in time. Near the left wall, an intense phase transition increases the vapor concentration, and the gas-phase filtration leads to some redistribution of the inert component (air). After complete evaporation of water, the porosity (curve 3 in Fig. 3) reaches its highest value $\alpha_{10} + \alpha_{20} = 0.3$ and remains unchanged afterwards. Owing to heat transfer between the solid phase and the gas, the gas temperature increases (see curve 2 in Fig. 2), and the vapor density decreases as a result of filtration.

Thus, the following typical regions can be distinguished in the profiles of parameters across the porous medium. In a vicinity of the left wall, water totally evaporates, and the highest temperature is observed here. The next zone is the water–vapor transition region [the inflection in the curve $T(x)$ (see Fig. 2)], in which the vapor concentration is maximum (see Fig. 3). The vapor transport to regions with lower temperatures results in condensation of water and a decrease in the porosity of the medium (see Fig. 3). Finally, as the x coordinate increases, the temperature approaches its initial value, the rise in pressure near the right wall being due to the increase in the air density. The spatial redistribution of air is caused by mass transfer.

REFERENCES

1. S. A. Khristianovich, "Fundamentals of the filtration theory," *Fiz. Tekh. Probl. Razrab. Polezn. Iskop.*, No. 1, 3–18 (1991).
2. D. V. Sadin, "Unsteady gas filtration," *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 1, 201–203 (1994).
3. R. I. Nigmatulin, *Dynamics of Multiphase Media*, Part 1, Hemisphere, New York (1991).
4. A. A. Samarskii, *Theory of Difference Schemes* [in Russian], Nauka, Moscow (1983).
5. D. V. Sadin, "Modified method of coarse particles for evaluating nonstationary gas flows in porous media," *Zh. Vychisl. Mat. Mat. Fiz.*, **36**, No. 10, 158–164 (1996).
6. P. L. Kirillov, Yu. S. Yur'ev, and V. P. Bobkov, *Handbook on Thermal and Hydraulic Design* (*Nuclear Reactors, Heat Exchangers, and Steam Generators*) [in Russian], Énergoatomizdat, Moscow (1990).